Lab 21

Helmholtz Coil

A. Purpose

To understand the structure of the Helmholtz coil and to study the magnetic field distribution created by the Helmholtz and anti-Helmholtz coils with the alternating current (AC) using a pick-up coil.

B. Introduction

The Helmholtz coil, named after the German physicist Hermann von Helmholtz, is a device for producing a region of a nearly uniform magnetic field. Fig. 1(a) shows the structure of the Helmholtz coil. The two current-carrying coils share the same central axis and have the same direction of the flowing currents. When the two coils are placed apart from each other by the distance of their radius, the on-axial magnetic field at the center of the coils happens to be the most uniform one, compared with that generated by the coils of different distances.

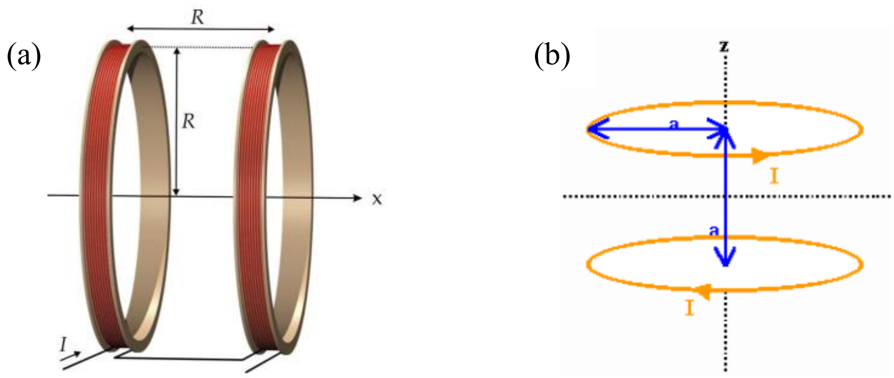


Fig. 1. Structure of (a) Helmholtz coil pair (b) Anti-Helmholtz coil pair

If the currents on the two coils have different directions, then the coil pair is called the anti-Helmholtz coil, as Fig. 1(b) shows. This construction makes a zero on-axial magnetic field and a linear magnetic field gradient at the center, which is usually applied to make a magneto-optic trap (MOT) in cold atom experiments.

The magnetic field generated by a Helmholtz coil pair is usually small. For example, the central magnetic field of a single coil in this lab is about 0.0024 Tesla (=24 Gauss), which is easily influenced by the background magnetic field (Earth, smartphones, etc.). Therefore, in this lab, we use an AC current input instead of a DC current input so that the effect of the background magnetic field can be eliminated during the experiment. To measure the magnetic field in the space, first, consider a coil with an AC current input. The oscillating current will generate an oscillating magnetic field around the coil. If there is another coil, called a pick-up coil, close to the oscillating magnetic field, based on Faraday's law, the induced electromotive force (EMF) will be created in the coil, by which one can measure the distribution of the magnetic field signal in the space. Usually, the pick-up coil signal is very weak. Therefore, the pick-up coil is connected in series with an LC resonance circuit to amplify the signal. The equivalent circuit of the pick-up coil and the LC resonance circuit is shown in Fig. 2. See Ref. 1 for more discussion on the pick-up coil.

The Helmholtz coil pair are also used in scientific apparatus to cancel external magnetic fields, usually the Earth's magnetic field. This lab studies the measurement of the magnetic field in the space by a pick-up coil with an LC resonance circuit as an amplifier and the magnetic fields generated by the one and two current-carrying coils. Note that in this lab, an inductor (formed by a coil with a ferrite core) serves as the inductor and the pick-up coil at the same time. The coil itself has a resistance, which makes the circuit an RLC circuit instead of just an LC circuit.

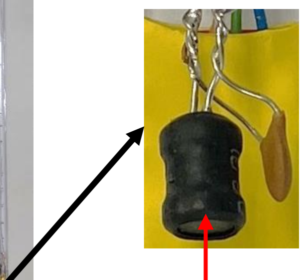
C. Apparatus

Oscilloscope

Function generator
(Output impedence: 600 Ω)

Two coils and base

Probe



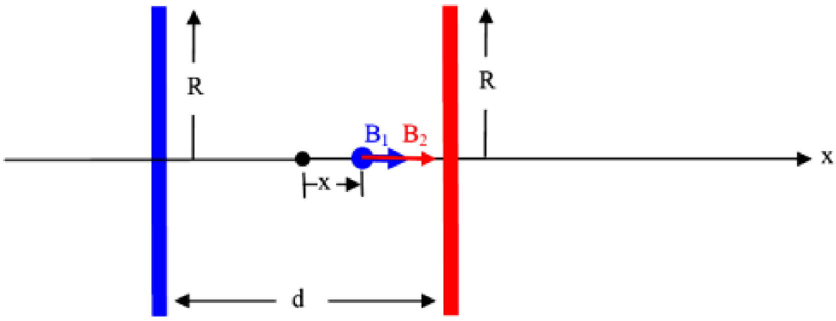
Capacitor
(0.1 μF)

Pick-up coil
(10 mH, 25 Ω)

Specifications of the current-carrying coil:		
Turns: 200	Inner radius: 10.06 cm	Outer radius: 10.58 cm	Coil width: 1.6 cm
Wire: Copper, 22 AWG (0.64 mm diameter)	Resistance: 7.2 Ω	Inductance: 16.3 mH
Maximum current: 2 A		

D. Procedures

1. Pre-lab assignments (hand in before the experiment)
   (1) Make a flowchart of this experiment and answer the questions.
   (2) Consider a coil pair as shown below. Suppose the center of the coil pair is the origin.



(i) Suppose there is a current I flowing in the two coils in the same direction. The two coils have the same number of turns N, and the coil pair is apart by a distance d. Find the formula of the on-axial magnetic fields $\mathbf{B}(\mathbf{x})$.

(ii) Use software to obtain the second derivative of the general on-axial magnetic field with respect to the coordinate, and prove that when the second derivative becomes zero, the coil pair becomes a Helmholtz coil pair, with the distance between the two coils having the same value of the radius of the coil R.

(iii) If the currents flowing in the two coils are in different directions, then the coil is called an anti-Helmholtz coil pair. Use software to prove that the gradient of the central magnetic field of an anti-Helmholtz coil pair is

$$\frac{dB}{dx} \approx 0.858 \frac{\mu_0 NI}{R^2}.$$

Also, find the positions of the extreme magnetic fields, and the magnitudes of the extreme magnetic fields.

(3) Suppose the center of the following coil (pair) is at the origin. Given that the current in the coil is $2\ \text{A}$, the radius of the coil is 10.5 cm, and the number of turns is 200. Use Matlab to draw the distribution of on-axial magnetic fields $\mathbf{B}(\mathbf{x})$ for $-15 \le x \le 15$ (cm).

(i) A single coil

(ii) A coil pair with the same direction of the flowing current. The distance between the two coils is

(a) 5 cm **(b)** 10.5 cm and **(c)** 15 cm.

(iii) A coil pair of the currents flowing in opposite directions. The distance between the two coils is 10.5 cm.

(4) Find the resonance frequency of the RLC circuit formed by the pick-up coil along with the capacitor in this lab. (Hint: Lab 19 RC and RLC circuits)

2. In-lab activities

(1) Pick-up coil

(i) Set up the experiment as Fig. 3 shows. **The pick-up coil should be at the center of the current-carrying coil.**

(ii) Set the input signal to be a sine wave with maximum amplitude and frequency of about 100 Hz. Use CH1 to show the signal of the pick-up coil and observe the signal across the external resistor of 100 ***Ω*** via CH2. The trigger source of the scope should be set to CH2. **(WHY?)** Take a photo as a record.

(iii) Change the frequency of the input sine wave so that the signal measured by the pick-up coil is the maximum. Take a photo as a record. (Hint: Pre-lab Q3)

(iv) Starting from the center of the current-carrying coil, use the probe to measure the peak-to-peak value V_{PP} of the signal obtained by the pick-up coil for the positions $d = -15 \sim 15$ cm. Change the distance d by 1 cm each time.

(v) Plot the peak-to-peak value V_{PP} versus the position d.

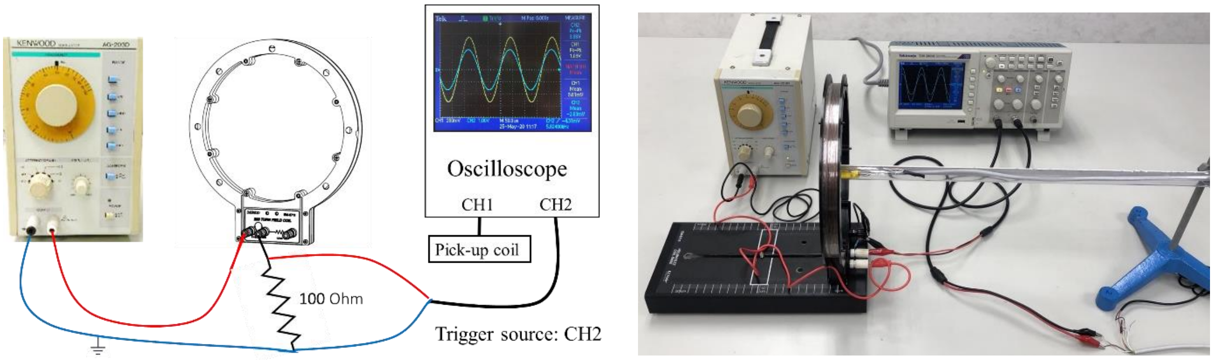


Fig. 3. Setup of pick-up coil experiment

(2) Helmholtz coil pair

(i) Following the setup of experiment 1, place another coil on the base to form a Helmholtz coil pair. The two coils should be screwed apart by 5 cm. Connect the two coils by the wires. Fig. 4 shows the experimental setup.

(ii) Starting from the center of the coil pair, use the probe to measure the peak-to-peak value V_{PP} of the signal obtained by the pick-up coil for the positions $d = -15 \sim 15$ cm. Change the distance d by 1 cm each time.

(iii) Plot the peak-to-peak value V_{PP} versus the position d.

(iv) Change the distance of the coil pair to 10.5 cm and 15 cm, respectively, and re-do steps (ii) and (iii). Take a photo of the central magnetic field signal of the Helmholtz configuration for a record.

(v) Compare the results of the three trials.

(3) Anti-Helmholtz coil pair

(i) Following the setup of experiment 2, screw the coil pair apart by 10.5 cm, and reverse the current direction of one coil by switching the connecting wires.

(ii) Repeat steps (ii) and (iii) in experiment 2.

(iii) Explain how to know the directions of the two extreme magnetic fields are opposite by the photos of the obtained signals.

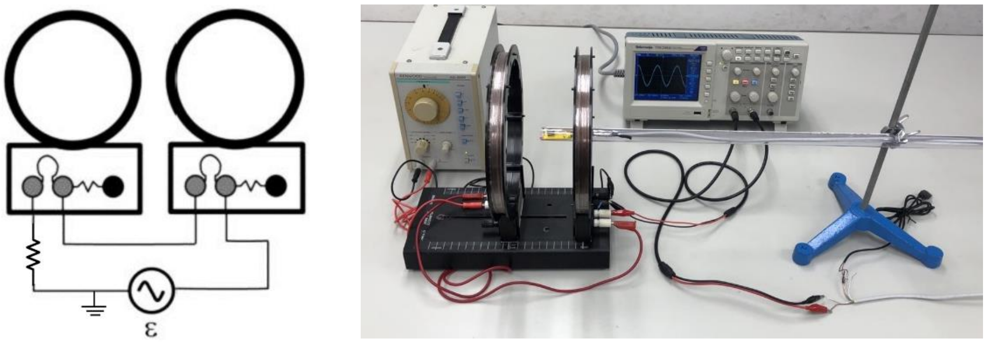


Fig. 4. Setup of Helmholtz coil experiment

3. Post-lab report
   (1) Recopy and organize your data from the in-lab tables in a neat and readable form.
   (2) Analyze the data you obtained in the lab and answer the given questions

E. Questions

1. Normalize the obtained magnetic field distributions by each maximum value, and compare the results with the normalized results of Matlab simulation.
2. For a Helmholtz coil pair, can we create a larger region of a nearly uniform magnetic field by increasing the radius of the coils and their distance at the same time? Explain by a physical picture.
3. Calculate the amplification factor of the pick-up coil in this lab. Compare your calculation with the photos in experiment 1. (Hint: the induced EMF is proportional to the first derivative of the flux in the pick-up coil.)
4. In experiment 1, it's observed that the signal of the magnetic field (CH1) and that of the current (CH2) are (roughly) in phase (or out of phase). Explain this phenomenon.
5. Compare the peak-to-peak value of the current signals of the single-coil, the Helmholtz coil pair, and the anti-Helmholtz coil pair. Explain their ordering. (Hint: Flux of the inductor.)
6. In the experiment, the Helmholtz coil pair is formed by two circular coils. If we replace the circular coils with the coils of other shapes, for example, two squares and two triangles, will the nearly uniform region of the magnetic field still be formed? Explain by a physical picture instead of the tedious mathematics.
7. **(Optional)** Based on the structure of the circular Helmholtz coil pair, propose a method to make a larger uniform region of the on-axial magnetic fields. For example, you can add one more circular coil to the original design.

F. References

Dietz, Eric R., and Robert W. Keith. "Inexpensive magnetic field mapping probe." The Physics Teacher 35.2 (1997): 112-115.